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EMPIRICAL PRICING KERNELS: EVIDENCE FROM THE HONG KONG STOCK MARKET

Abstract. *In this paper, we investigate the empirical pricing kernels for the Hong Kong stock market. We deal with semiparametric estimation of the empirical pricing kernel as the ratio of the objective and risk-neutral densities, under a consistent parametric framework of the non-affine GARCH diffusion model. An efficient importance sampling (EIS)-based joint maximum likelihood estimation method is developed for the objective and risk-neutral densities, using the Hang Seng Index (HSI) and index warrants data. Empirical results show that there exists a reference point and around this reference point the empirical pricing kernel exhibits a hump. The market utility function does not correspond to standard specification of utility function in the classical expected utility theory, but exhibits a convex form below the reference point and a concave form above it, and the investors act risk seeking around the reference point.*

Keywords: *pricing kernel; utility function; risk aversion; GARCH diffusion model; maximum likelihood estimation.*

JEL Classification: C13, C32, C58, G13

1. Introduction

The behaviour of market investors has always been in focus in the literature on financial economics. Naturally, it involves the empirical pricing kernel (Rosenberg and Engle, 2002). The asset pricing kernel contains a wealth of information, which summarizes the pattern of the market utility function and investor risk preference.

In standard economic theory, the pricing kernel is a monotonically decreasing function of the market return, corresponds to a concave utility function and investor risk aversion. However, there has been a lot of discussion about the reliability of this theory. In particular, several recent empirical studies showed that there is a reference point near the zero return and around this reference point the empirical pricing kernel exhibits a hump (see e.g., Jachwerth, 2000; Detlefsen et al., 2007). Hence, the investors act risk seeking around the reference point. The non-monotonicity of the empirical pricing kernel has become known as the "pricing kernel puzzle" or "risk aversion puzzle". Numerous attempts have been undertaken to explain the reason for the pricing kernel puzzle from different perspectives (see e.g., Detlefsen et al., 2007; Ziegler, 2007; Chabi-Yo et al., 2008; Bakshi et al., 2010; Gollier, 2011; Chabi-Yo, 2012; Christoffersen et al., 2013; Hens and Reichlin, 2013; Barone-Adesi et al., 2015, and among many others). On the other hand, Beare and Schmidt (2014) and Golubev et al. (2014) find the evidence of non-monotonically decreasing pricing kernel by conducting formal statistical test about the shape of the pricing kernel. Their results provide empirical support for the financial economics literature on the pricing kernel puzzle.

In the last decades, there is a large literature on the estimation of the pricing kernel. A number of earlier papers estimate the pricing kernel using aggregate consumption data (see e.g., Hansen and Jagannathan, 1991; Chapman, 1997), problems with imprecise measurement of aggregate consumption can weaken the empirical results of these papers. Recently, many authors have used the historical returns and option prices data to estimate the pricing kernel. This approach avoids the use of aggregate consumption data. Based on the returns and option prices data, three types of estimation approaches for estimating the pricing kernel have been developed: parametric approaches (e.g., Rosenberg and Engle, 2002), nonparametric approaches (e.g., Aït-Sahalia and Lo, 2000; Jackwerth, 2000; Song and Xiu, 2016) and semiparametric approaches (e.g., Chernov, 2003; Detlefsen et al., 2007).

However, the parametric approaches which impose a strict structure on the kernel are too restrictive to account for the dynamics of the risk preference, while the nonparametric approaches depend a lot on the bandwidth selection which influences the shape of the pricing kernel. The semiparametric approaches avoid the use of parametric pricing kernel specification and bandwidth selection, which is flexible and simple to implement. Therefore, we dervie the empirical pricing kernel in this paper by employing a semiparametric approach based on the objective and risk-neutral densities. Previous econometrics studies are concerned with deriving the empirical pricing kernel by estimating the objective and risk-neutral densities separately, and relying on the discrete-time GARCH model or/and Heston (1993) model. Our estimation procedure is based on the objective and risk-neutral densities and these distributions are derived jointly with a consistent parametric stochastic volatility framework of non-affine GARCH diffusion model. From these densities we construct the corresponding pricing kernel. The GARCH diffusion model is a non-affine stochastic volatility model, which has been found to capture the dynamics of the financial time series much better than the popular affine stochastic volatility model of Heston (1993). Moreover, a number of recent papers have provide strong evidence for the GARCH diffusion model not only for returns data but also for options data (e.g., Christoffersen et al., 2010; Wu et al., 2012; Kaeck and Alexander, 2013). Thus, the model is well suited for our estimation of the pricing kernel.

The objective and risk-neutral densities are derived by estimating jointly the objective and risk-neutral parameters of the GARCH diffusion model. In this paper, we develop an joint estimation procedure for estimating the model using the Hong Kong Hang Seng Index (HSI) and index warrant prices data. The fundamental advantage of this approach is that all the parameters of the model can be reliably identified in a way that maintains the internal consistency of the objective and risk-neutral measures. The joint estimation procedure we adopt in this paper is based on the maximum likelihood method where the likelihood function is evaluated using the efficient importance sampling (EIS) technique of Richard and Zhang (2007). The EIS-based joint maximum likelihood method is easy to implement and enables us to estimate the parameters of the GARCH diffusion model efficiently.

The rest of the paper is organized as follows. In Section 2, we describe the theoretical relationship between the pricing kernel, market utility function and absolute risk aversion and the objective and risk-neutral densities. In Section 3, we present under both the objective and risk-neutral measures the GARCH diffusion model, which serves as the basis for the estimation of the objective and risk-neutral densities, and discuss how to estimate jointly the objective and risk-neutral parameters of the GARCH diffusion model using data on the HSI returns and index warrant prices. In Section 4, we discuss the empirical pricing kernels obtained from the HSI data, and we conclude in Section 5. Technical details are provided in appendices to the paper.

2. Pricing kernel, market utility function and absolute risk aversion

In the absence of arbitrage, there exists one positive random variable $M_{t,T}$ such that the current price P_t of an asset with payoff ψ_T at time T is

$$P_t = E^{\mathrm{P}}[M_{t,T}(X_T)\psi_T(X_T) \mid \mathrm{F}_t] \tag{1}$$

where X_T is the state variable of the economy (e.g., log aggregate consumption), E^{P} is the expectation with respect to the objective measure P, $M_{t,T}$ is called the pricing kernel, and F_t is the information up to and including time t.

According to the risk-neutral valuation principal, the price P_t of the asset can be equivalently represented as

$$P_t = E^{¤}[e^{-r\tau}\psi_T(X_T) \mid \mathrm{F}_t] \tag{2}$$

where $E^{¤}$ is the expectation with respect to the risk-neutral measure ¤, r is the risk free interest rate, $\tau = T - t$. Assuming that $p_{t,T}(X_T)$ and $q_{t,T}(X_T)$ are the objective density and risk-neutral density of X_T, respectively. From Eq. (2), we have

$$P_t = \int_{\mathbb{R}} e^{-r\tau}\psi_T(x)q_{t,T}(x)dx = \int_{\mathbb{R}} e^{-r\tau}\psi_T(x)\frac{q_{t,T}(x)}{p_{t,T}(x)}p_{t,T}(x)dx$$
$$= E^{\mathrm{P}}\left[e^{-r\tau}\psi_T(X_T)\frac{q_{t,T}(X_T)}{p_{t,T}(X_T)} \mid \mathrm{F}_t\right] \tag{3}$$

Compare Eqs. (1) and (3), we get

$$M_{t,T}(X_T) = e^{-r\tau}\frac{q_{t,T}(X_T)}{p_{t,T}(X_T)} \tag{4}$$

In a dynamic equilibrium model, the pricing kernel is equal to the intertemporal marginal rate of substitution, i.e.,

$$M_{t,T}(X_T) = \frac{U'(X_T)}{U'(X_t)} \tag{5}$$

Here the state variable, X_T, is log aggregate consumption, which can be substituted with log equity index or equity index return (e.g., Rosenberg and Engle, 2002). Thus, from Eqs. (4) and (5), we have

$$e^{-r\tau}\frac{q_{t,T}(X_T)}{p_{t,T}(X_T)} = \frac{U'(X_T)}{U'(X_t)} \tag{6}$$

Then we can derive the market utility function as

$$U(X_T) = U(X_t) + e^{-r\tau}U'(X_t)\int_{X_t}^{X_T}\frac{q_{t,T}(x)}{p_{t,T}(x)}dx = U(X_t) + U'(X_t)\int_{X_t}^{X_T}M_{t,T}(x)dx \tag{7}$$

Besides the pricing kernel and market utility function, we are also interested in the investor risk preference in the market. Such risk preference is often described in terms of Arrow-Pratt measure of absolute risk aversion that is define by

$$ARA(X_T) = -\frac{U''(X_T)}{U'(X_T)} \tag{8}$$

From Eq. (6), we get

$$U'(X_T) = e^{-r\tau}U'(X_t)\frac{q_{t,T}(X_T)}{p_{t,T}(X_T)} \tag{9}$$

and

$$U''(X_T) = e^{-r\tau}U'(X_t)\frac{q'_{t,T}(X_T)p_{t,T}(X_T) - q_{t,T}(X_T)p'_{t,T}(X_T)}{p^2_{t,T}(X_T)} \quad (10)$$

Plugging Eqs. (9) and (10) into Eq. (8), we get the absolute risk aversion in terms of the objective and risk-neutral densities:

$$\begin{aligned} ARA(X_T) &= -\frac{e^{-r\tau}U'(X_t)(q'_{t,T}(X_T)p_{t,T}(X_T) - q_{t,T}(X_T)p'_{t,T}(X_T)) / p^2_{t,T}(X_T)}{e^{-r\tau}U'(X_t)q_{t,T}(X_T) / p_{t,T}(X_T)} \\ &= \frac{p'_{t,T}(X_T)}{p_{t,T}(X_T)} - \frac{q'_{t,T}(X_T)}{q_{t,T}(X_T)} \end{aligned} \quad (11)$$

3. Estimation methodology

We adopt the non-affine GARCH diffusion model to characterize the dynamics of the HSI index, and form the basis for the estimation of the objective and risk-neutral densities. We first describe the model under the objective and risk-neutral measures in Section 3.1, and then discuss how to estimate jointly the objective and risk-neutral parameters of the GARCH diffusion model using data on the HSI returns and index warrant prices in Section 3.2. Additional informations about likelihood approximation and unobservable state variables estimation are given in Appendices A and B.

3.1 The model

In the GARCH diffusion model, the dynamics under the objective measure of the HSI index price S_t and the associated volatility V_t are assumed to be given by

$$dS_t = \mu S_t dt + \sqrt{V_t}S_t dW_{1t} \quad (12)$$

$$dV_t = (\alpha - \beta V_t)dt + \sigma V_t dW_{2t} \quad (13)$$

where μ is the mean of the HSI returns, α / β is the long-run mean of volatility, β is the mean reversion rate of volatility, σ is the volatility of volatility, and W_{1t} and W_{2t} are two standard Brownian motions with $\mathrm{Corr}_t(dW_{1t}, dW_{2t}) = \rho$.

Similar to Chernov and Ghysels (2000), we assume that the GARCH diffusion model have the same form under the risk-neutral measure as under the objective measure, and the dynamics of (S_t, V_t) under the risk-neutral measure are of the form

$$dS_t = rS_t dt + \sqrt{V_t}S_t dW^*_{1t} \quad (14)$$

$$dV_t = (\alpha^* - \beta^* V_t)dt + \sigma V_t dW^*_{2t} \quad (15)$$

where r is the risk-free interest rate, W_{1t}^* and W_{2t}^* are two standard Brownian motions under the risk-neutral measure with $\mathrm{Corr}_t(dW_{1t}^*, dW_{2t}^*) = \rho$.

Following Wu et al. (2012), the characteristic function for the log HSI index $X_T = \ln S_T$ can be derived. Then the objective/risk-neutral densitiy for X_T can be obtained by inverting the corresponding characteristic function. That is,

$$p_{t,T}(X_T) = \frac{1}{2\pi}\int_{\text{i}} e^{-\mathbf{i}\phi X_T} f_{t,T}(\phi)d\phi \tag{16}$$

$$q_{t,T}(X_T) = \frac{1}{2\pi}\int_{\text{i}} e^{-\mathbf{i}\phi X_T} f_{t,T}^*(\phi)d\phi \tag{17}$$

where $f_{t,T}$ and $f_{t,T}^*$ are the characteristic functions for X_T under the objective and risk-neutral measures, respectively, and the integrals in Eqs. (16) and (17) can be easily computed by using some numerical methods.

3.2 Joint maximum likelihood estimation

In this subsection, we develop a maximum likelihood method to estimate jointly the objective and risk-neutral parameters of the GARCH diffusion model using data on the HSI returns and index warrant prices.

Taking the stabilizing transformation $X_t = \ln S_t$, $h_t = \ln V_t$. By Itô's lemma, we have

$$dX_t = (\mu - \frac{1}{2}e^{h_t})dt + e^{h_t/2}dW_{1t} \tag{18}$$

$$dh_t = (\alpha e^{-h_t} - \beta - \frac{1}{2}\sigma^2)dt + \sigma dW_{2t} \tag{19}$$

In the empirical literature, the above continuous-time model must be discretized to facilitate the parameter estimation. A simple Euler scheme leads to the following discrete-time stochastic processes

$$y_{t_i} = (\mu - \frac{1}{2}e^{h_{t_{i-1}}})\Delta_i + e^{h_{t_{i-1}}/2}\sqrt{\Delta_i}\varepsilon_i \tag{20}$$

$$h_{t_i} = h_{t_{i-1}} + (\alpha e^{-h_{t_{i-1}}} - \beta - \frac{1}{2}\sigma^2)\Delta_i + \sigma\sqrt{\Delta_i}\eta_i \tag{21}$$

where $y_{t_i} = X_{t_i} - X_{t_{i-1}}$ is the HSI return, $\Delta_i = t_i - t_{i-1}$ is the time interval, ε_i and η_i are independent and identically distributed (i.i.d.) standard normal random variables with $\mathrm{Corr}_t(\varepsilon_i, \eta_i) = \rho$.

To perform joint estimation of the objective and risk-neutral parameters, we consider the additional information provided by the HSI warrant prices. We assume that the observed warrant price is equal to the theoretical value plus a pricing error:

$$C_{t_i} = C(t_i, \tau_i, K, S_{t_i}, V_{t_i}) + \delta v_i \tag{22}$$

where the nonlinear function $C(t_i, \tau_i, K, S_{t_i}, V_{t_i})$ is the pricing formula for European warrants in the GARCH diffusion model (see Wu et al., 2012), and v_i are i.i.d. standard normal random variables and independent of ε_i and η_i.

It is obvious that Eqs. (20)-(22) constitute a nonlinear and non-Gaussian state-space model with volatility is the unobservable state variable. To estimate this model using maximum likelihood method, we need to integrate out the unobservable state variables from the joint density of the observations and unobservable state variables and derive an explicit expression for the marginal likelihood of observations.

Let $C = (C_{t_1}, \ldots, C_{t_N})'$ be a vector of the N observed HSI index warrant prices, $Y = (y_{t_1}, \ldots, y_{t_N})'$ be a vector of the N observed HSI returns and $H = (h_{t_1}, \ldots, h_{t_N})'$ be a vector of the unobservable state variables (log volatilities). The likelihood function of the model can be expressed as

$$\mathrm{L}(C, Y; \Theta, h_{t_0}) = \int p(C, Y, H; \Theta, h_{t_0}) dH \tag{23}$$

where $\Theta = (\mu, \alpha, \beta, \sigma, \rho, \alpha^*, \beta^*, \delta)$ is the parameter vector, which consists of the objective and risk-neutral parameters $(\mu, \alpha, \beta, \sigma, \rho, \alpha^*, \beta^*)'$ of the GARCH diffusion model and the parameter δ in measurement equation (22), and $p(C, Y, H; \Theta, h_{t_0})$ is the joint density of C, Y and H, which can be written as

$$\begin{aligned} p(C, Y, H; \Theta, h_{t_0}) &= p(C \mid Y, H, \Theta) p(Y, H; \Theta, h_{t_0}) \\ &= \prod_{i=1}^{N} p(C_{t_i} \mid y_{t_i}, h_{t_i}, \Theta) p(y_{t_i} \mid h_{t_{i-1}}, \Theta) p(h_{t_i} \mid y_{t_i}, h_{t_{i-1}}, \Theta) \end{aligned} \tag{24}$$

where $p(C_{t_i} \mid y_{t_i}, h_{t_i}, \Theta)$ is the normal density of C_{t_i} with the conditional mean $C(t_i, \tau_i, K, S_{t_i}, V_{t_i})$ and the conditional variance δ^2, $p(y_{t_i} \mid h_{t_{i-1}}, \Theta)$ is the normal density of y_{t_i} with the conditional mean $(\mu - \frac{1}{2} e^{h_{t_{i-1}}})\Delta_i$ and the conditional variance $e^{h_{t_{i-1}}} \Delta_i$ and $p(h_{t_i} \mid y_{t_i}, h_{t_{i-1}}, \Theta_i)$ is the normal density of h_{t_i} with the conditional mean μ_{t_i} and the conditional variance $\sigma_{t_i}^2$ which are given by

$$\mu_{t_i} = h_{t_{i-1}} + (\alpha e^{-h_{t_{i-1}}} - \beta - \frac{1}{2}\sigma^2)\Delta_i + \sigma\rho \frac{y_{t_i} - (\mu - \frac{1}{2} e^{h_{t_{i-1}}})\Delta_i}{e^{h_{t_{i-1}}/2}} \tag{25}$$

$$\sigma_{t_i}^2 = \sigma^2(1-\rho^2)\Delta_i \tag{26}$$

Given the likelihood function in Eq. (23), the ML estimates of parameters of the state-space model in Eqs. (20)-(22) are then given by

$$(\hat{\Theta}, \hat{h}_{t_0}) = \arg\max_{(\Theta, h_{t_0})} \ln \mathrm{L}(C, Y; \Theta, h_{t_0})$$

As a typical financial time series has at least several hundreds of observations, the high-dimensional integral in the right hand of Eq. (23) rarely has analytical expression. Meanwhile, using the traditional numerical integration methods to approximate the integral is also infeasible. In order to overcome this problem, we adopt the EIS technique to compute the likelihood function. The EIS algorithm for likelihood approximation is presented in Appendix A. To extract the latent spot volatility, we use a particle filter algorithm which is given in Appendix B.

4. Empirical analysis

In contrast to many previous studies that have focused mainly on the S&P 500 data, we investigate in this paper the empirical pricing kernels by focusing on the HSI data (HSI index and its warrants). The HSI index serves as an approximation to the Hong Kong economy, and it can be used as a proxy for market portfolio. The HSI index warrants were chosen over the HSI index options because the HSI warrants market is a more liquid/active market than the HSI options market in Hong Kong.

4.1 The data

In the empirical analysis we use daily data on the HSI returns and index warrant prices from July 21, 2011 to May 31, 2013. The HSI returns computed are logarithmic, i.e., $x_t = \log p_t - \log p_{t-1}$, where p_t is the closing price. The HSI index warrant is chosen as the HS-HSI@EC1309 which is one of the most actively traded HSI index warrants. The selected warrant is European-style call warrant which is similar to a European-style call option. Its maturity date is September 27, 2013, the exercise price is 25,000 and the exercise ratio is 12,000. The sample size is 918 for the joint data. The time-series of HSI returns and HS-HSI@EC1309 prices are plotted in Figure 1. Finally, we use the 1-year Hong Kong Interbank Offer Rate (HIBOR) as a proxy for the risk-free interest rate. All of the data are obtained from the Wind Database of China.

Summary statistics for the HSI returns are shown in Table 1. As can be seen from the table, the HSI returns are skewed and leptokurtic. Jarque-Bera statistics suggests that the assumption of normality is rejected for the HSI return series. Furthermore, from Figure 1 we can observe that the HSI returns exhibit time-varying volatility and volatility clustering during the sample period.

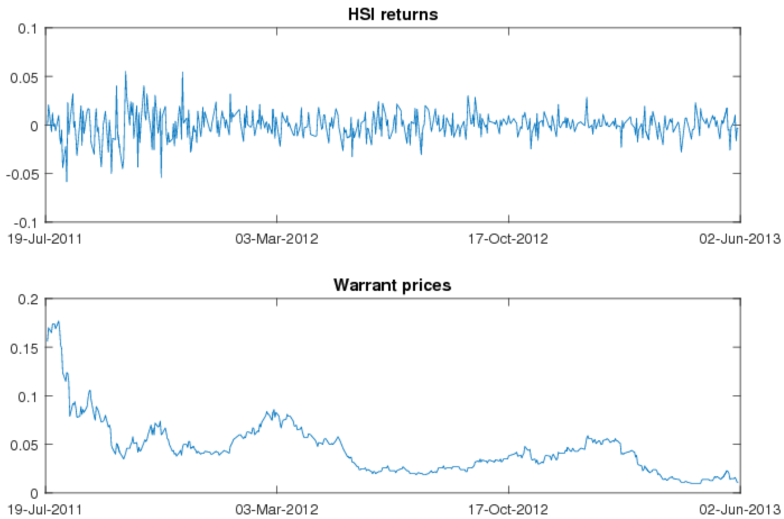

Figure 1: Time series of HSI returns and HS-HSI@EC1309 prices for the sample period from July 21, 2011 to May 31, 2013

Table 1. Summary statistics of HSI returns

Mean	Max	Min	Std.	Skew	Kurt	Jarque-Bera
0.0000	0.0552	-0.0583	0.0137	-0.2840	5.6931	144.875 (0.000)

Note: The number in parenthesis is the p-values of Jarque-Bera tests.

4.2 Estimation results

Based upon data on the HSI returns and HS-HSI@EC1309 prices, the objective and risk-neutral parameters of the GARCH diffusion model are estimated jointly by applying the maximum likelihood method described in Section 3. Table 2 reports the estimation results.

The estimated parameters allow us to estimate the volatility, V_t, via the particle filter algorithm. The number of particles used in the empirical studies is 1000. Figure 2 plots the estimated volatilities.

Table 2. Estimation results

μ	α	β	σ	ρ
0.2447 (0.0730)	0.1947 (0.0507)	2.3710 (1.5585)	1.2780 (0.0175)	-0.5373 (0.0155)
α^*	β^*	δ	**Log-lik**	
0.0321 (0.0076)	0.8001 (0.0231)	0.0004 (0.0000)	3515.85	

Note: The EIS-ML method is implemented by using S=32 Monte Carlo draws and 5 EIS iterations. Log-lik is the log-likelihood value. The number in parenthesis is the standard error.

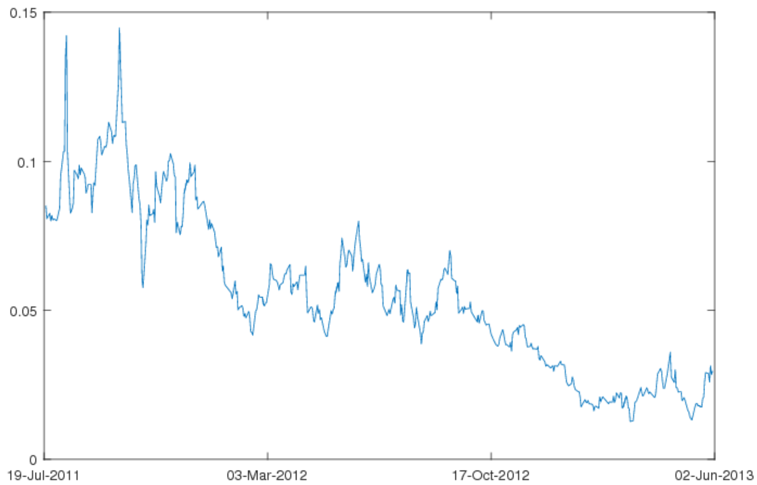

Figure 2: Estimated volatilities

Based on the estimates of the objective and risk-neutral parameters and volatilities, the objective and risk-neutral densities of the HSI returns can be obtained by using Eqs. (16) and (17). The estimation results of the objective and risk-neutral densities are presented in Figure 3 for the day May 31, 2013 and for two time to maturities: $\tau = 0.5$ and 1 years. It can be seen that there are large discrepancies in the estimation results of the objective and risk-neutral densities.

By using the Eqs. (4), (7) and (11), we derive the estimated empirical pricing kernels, market utility functions and absolute risk aversion functions of HSI returns on May 31, 2013 for two time to maturities: $\tau = 0.5$ and 1 years, which are presented in Figures 4-6. As can be seen from Figure 4, our estimated empirical pricing kernels are not monotonically decreasing, and these are not in accordance with the classical economic theory. The estimated empirical pricing kernels have humps located at small losses (corresponding to a HSI return of about -10% for time to maturity $\tau = 0.5$ and a HSI return of about -12% for time to maturity $\tau = 1$, hereafter referred to as reference points). Our results provide empirical support for the literature on the pricing kernel puzzle.

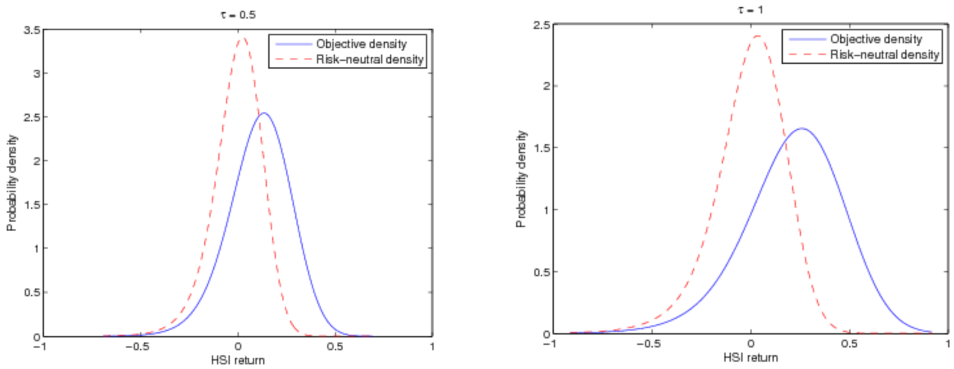

Figure 3: Estimated objective and risk-neutral densities on May 31, 2013 for time to maturities $\tau = 0.5$ and 1 years

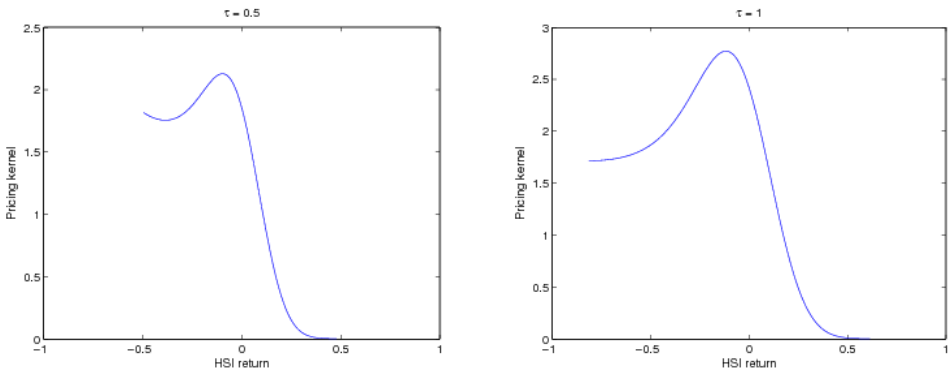

Figure 4: Empirical pricing kernels on May 31, 2013 for time to maturities $\tau = 0.5$ and 1 years

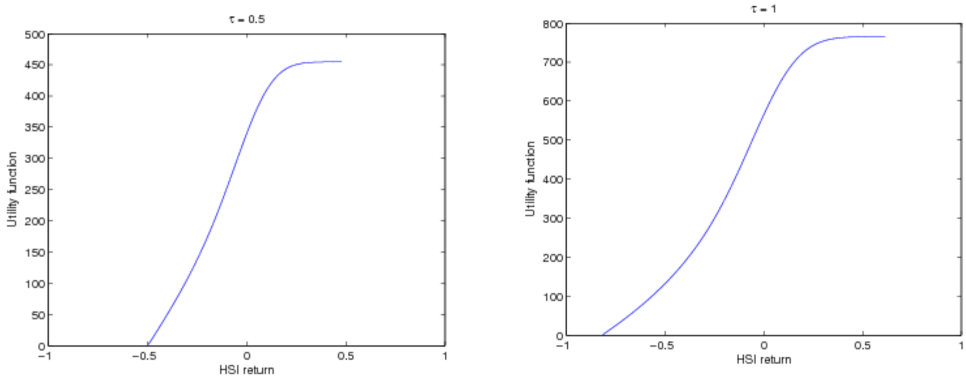

Figure 5: Market utility functions on May 31, 2013 for time to maturities $\tau = 0.5$ and 1 years

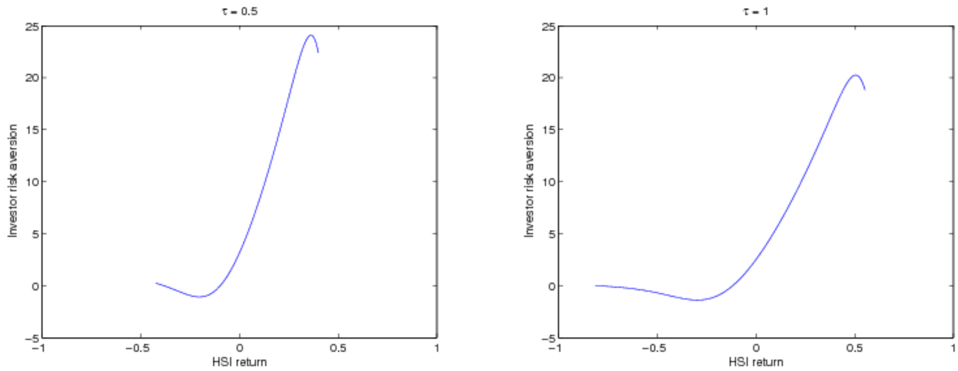

Figure 6: Absolute risk aversion functions on May 31, 2013 for time to maturities $\tau = 0.5$ and 1 years

The pricing kernels are the link between the absolute risk aversions and the market utility functions that are presented in Figure 5. As can be seen from the figure, the estimated market utility functions are increasing but do not correspond to standard specification of utility function in the classical expected utility theory. Specifically, the estimated market utility function exhibits a convex form below the reference point and a concave form above it, which is in accordance with the utility function form proposed by Kahneman and Tversky (1979).

Finally, we consider the absolute risk aversions in the Hong Kong stock market. The estimated absolute risk aversion functions are presented in Figure 6. It can be seen from the figure that the absolute risk aversion is negative around the reference point, which implies that investors act risk seeking around the reference point. Our results are much in line with the prospect theory of Kahneman and Tversky (1979).

5. Conclusion

In this paper, we employ a semiparametric approach to derive the empirical pricing kernels as the ratio of the objective and risk-neutral densities for the Hong Kong stock market. The objective and risk-neutral densities are estimated jointly by the maximum likelihood method based on the EIS technique, under a consistent parametric framework of the non-affine GARCH diffusion model and using the HSI returns and index warrant prices data. Empirical results show that there exists a reference point (corresponding to a HSI return of about -10%/-12% for half-year/one-year maturity) and around this reference point the empirical pricing kernel exhibits a hump. The market utility function does not correspond to standard specification of utility function in the classical expected utility theory, but exhibits a convex form below the reference point and a concave form above it, and the investors act risk seeking around the reference point. Our results are much in line with the prospect theory of Kahneman and Tversky (1979) and provide empirical support for the literature on the pricing kernel puzzle.

Acknowledgements

This work was supported by the National Natural Science Foundation of China under Grant No. 71501001, the MOE (Ministry of Education in China) Project of Humanities and Social Sciences under Grant No. 14YJC790133, the China Postdoctoral Science Foundation under Grant No. 2015M580416, the Natural Science Foundation of Anhui Province of China under Grant No. 1408085QG139, and the Anhui Province College Excellent Young Talents Fund of China under Grant No. 2013SQRW025ZD.

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Appendix A. EIS algorithm to likelihood approximation

The EIS algorithm for estimating the likelihood function is given as follows:

Step 1: Draw initial samples $\{h_{t_1}^{(s)},\ldots,h_{t_N}^{(s)}\}_{s=1}^{S}$ from the so-called natural importance sampler $\{p(h_{t_i}\mid y_{t_i},h_{t_{i-1}},\Theta)\}_{i=1}^{N}$.

Step 2: Calculate $\hat{a}_{t_i}$ by estimating the following regression model (working backwards: $t_i=t_N\to t_1$)

$$\ln p(C_{t_i}\mid y_{t_i},h_{t_i}^{(s)},\Theta)+\ln p(y_{t_{i+1}}\mid h_{t_i}^{(s)},\Theta)+\ln\chi_{t_{i+1}}(y_{t_{i+1}},h_{t_i}^{(s)},\hat{a}_{t_{i+1}})$$
$$=c_{t_i}+a_{1,t_i}h_{t_i}^{(s)}+a_{2,t_i}(h_{t_i}^{(s)})^2+u_{t_i}^{(s)},\ \ s=1,\ldots,S$$

where $\ln\chi_{t_i}(y_{t_i},h_{t_{i-1}},a_{t_i})=\dfrac{1}{2}\ln\dfrac{\sigma_{a_{t_i}}^2}{\sigma_{t_i}^2}+\dfrac{\mu_{a_{t_i}}^2}{2\sigma_{a_{t_i}}^2}-\dfrac{\mu_{t_i}^2}{2\sigma_{t_i}^2}$, $\mu_{a_{t_i}}=\sigma_{a_{t_i}}^2\left(a_{1,t_i}+\dfrac{\mu_{t_i}}{\sigma_{t_i}^2}\right)$,

$\sigma_{a_{t_i}}^2=\dfrac{\sigma_{t_i}^2}{1-2a_{2,t_i}\sigma_{t_i}^2}$, $p(y_{t_{N+1}}\mid h_{t_N},\Theta)\equiv\chi_{t_{N+1}}(y_{t_{N+1}},h_{t_N},\hat{a}_{t_{N+1}})\equiv 1$, $a_{t_i}=(a_{1,t_i},a_{2,t_i})$,

μ_{t_i} and $\sigma_{t_i}^2$ are given in Eqs. (25) and (26).

Step 3: Draw new samples $\{h_{t_1}^{(s)},\ldots,h_{t_N}^{(s)}\}_{s=1}^{S}$ from the EIS sampler $\{m_{t_i}(h_{t_i}\mid y_{t_i},h_{t_{i-1}},\hat{a}_{t_i})\}_{i=1}^{N}$, where m_{t_i} is the normal density (called EIS density) of h_{t_i} with the conditional mean $\mu_{a_{t_i}}$ and the conditional variance $\sigma_{a_{t_i}}^2$.

Step 4: Repeat Step 2 and Step 3, until a reasonable convergence of the parameters $\hat{a}_{t_i}$ is obtained.

Step 5: Calculate the likelihood approximation using

$$\tilde{L}(C,Y;\Theta,h_0)=\frac{1}{S}\sum_{s=1}^{S}\left[\prod_{i=1}^{N}\frac{p(C_{t_i}\mid y_{t_i},h_{t_i}^{(s)},\Theta)\,p(y_{t_i}\mid h_{t_{i-1}}^{(s)},\Theta)\,p(h_{t_i}^{(s)}\mid y_{t_i},h_{t_{i-1}}^{(s)},\Theta)}{m_{t_i}(h_{t_i}^{(s)}\mid y_{t_i},h_{t_{i-1}}^{(s)},\hat{a}_{t_i})}\right]$$

Following Richard and Zhang (2007), a same set of Common Random Numbers (CRNs) is used to obtain the draws from the EIS sampler in order to ensure the likelihood approximation be a smooth function of the parameter vector. Typically, a reasonable convergence can be obtained after 3-5 iterations.

Appendix B. Particle filter algorithm for extracting latent state variables

The particle filter algorithm for extracting the latent state variables is given as follows:

Step 1: Given a set of random samples $\{h_{t_{i-1}}^{(1)},\ldots,h_{t_{i-1}}^{(M)}\}$ from the probability density function $p(h_{t_{i-1}}\mid \mathrm{F}_{t_{i-1}})$.

Step 2: Draw samples $\{h_{t_i}^{(1*)},\ldots,h_{t_i}^{(M*)}\}$ from the probability density $p(h_{t_i}\mid h_{t_{i-1}},\Theta)$.

Step 3: Compute the normalised weight for each sample

$$q_j=\frac{p(C_{t_i}\mid y_{t_i},h_{t_i}^{(j*)},\Theta)\,p(y_{t_i}\mid h_{t_i}^{(j*)},h_{t_{i-1}}^{(j)},\Theta)}{\sum_{l=1}^{M}p(C_{t_i}\mid y_{t_i},h_{t_i}^{(l*)},\Theta)\,p(y_{t_i}\mid h_{t_i}^{(l*)},h_{t_{i-1}}^{(l)},\Theta)},\quad j=1,\ldots,M$$

Thus define a discrete distribution over $\{h_{t_i}^{(1*)},\ldots,h_{t_i}^{(M*)}\}$, with probability mass $\{q_1,\ldots,q_M\}$.

Step 4: Resample M times from the discrete distribution defined above to generate samples $\{h_{t_i}^{(1)},\ldots,h_{t_i}^{(M)}\}$.